Organising Compositionality in Classical Mechanics

Category Theory October/Novemberfest 2026

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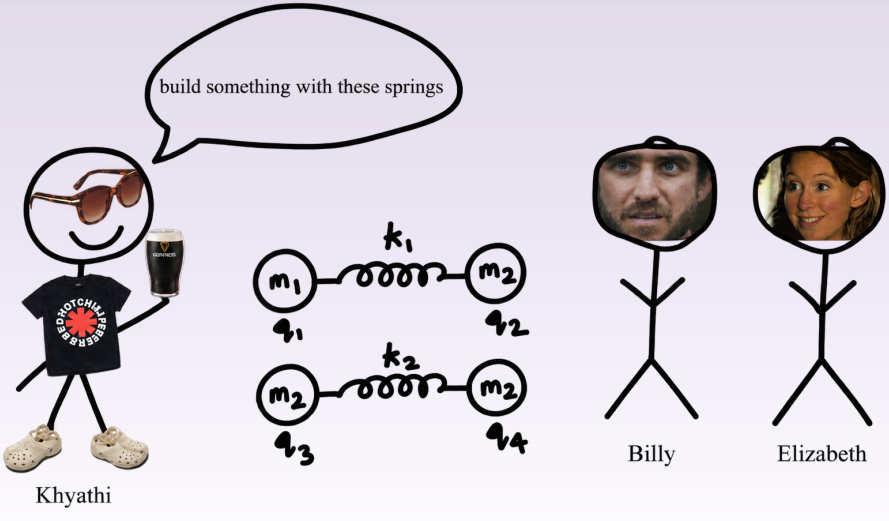
Joint Work with David Jaz Myers

October 31, 2026

Can we compose classical mechanical systems?
Can we separate the parts of a mechanical system from the choice of how they are connected?

- Two springs, how may I connect them?
- Lagrangian and Hamiltonian descriptions of classical mechanics
- The Baez-Weisbart-Yassine approach and its limitations
- Our loose action construction
- Examples: One spring, two springs side by side, and two springs sharing a mass
- Motivating future work

I. A Tale of Two Springs



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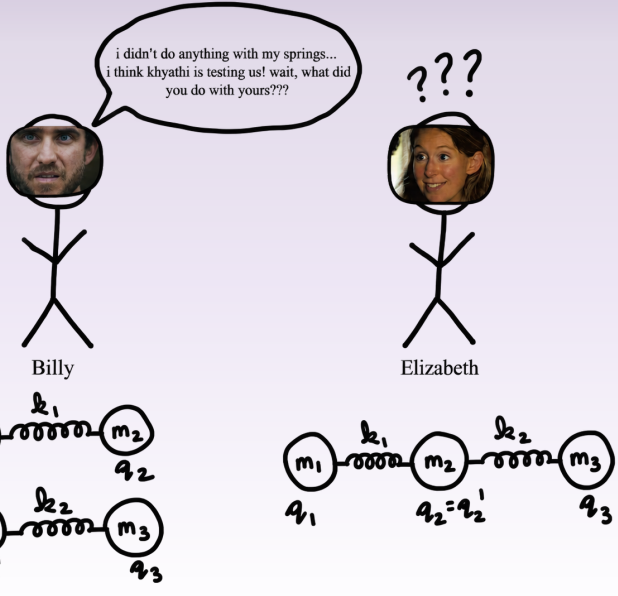

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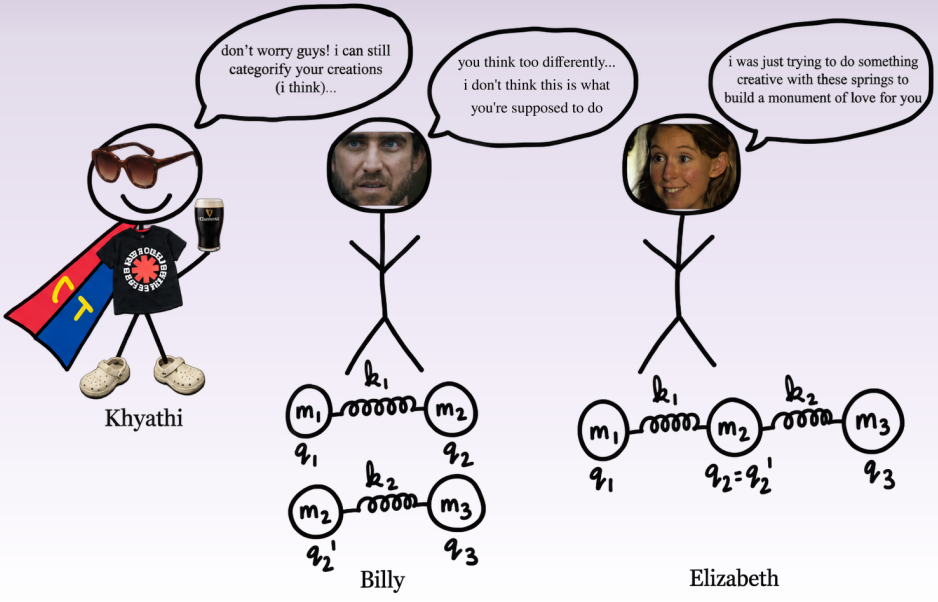
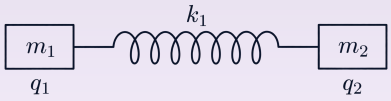


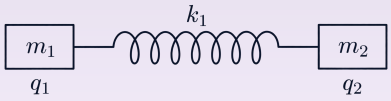


Parameters
The masses m_1, m_2 and spring constant k_1 specify the physical ingredients.

Configuration
A point $q = (q_1, q_2) \in Q = \mathbb{R}^2$ encodes where the two masses are.

State
Velocity states live in TQ, momenta lives in T^*Q carrying the symplectic form $\omega_{\text{can}} = \sum_i dq_1 \wedge dp_i$

Dynamics
Finally, we need a function telling us how these states evolve.



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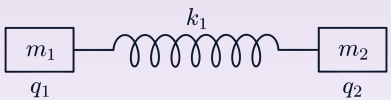

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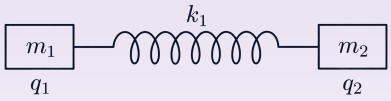

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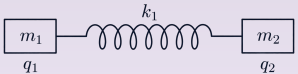


Stretching the spring stores potential energy: $V = \frac{1}{2}k_1(q_1 - q_2)^2$.

Lagrangian mechanics

Describe the state using positions and velocities: $(q, v) \in TQ$.

$L = T - V$: motion contributes positively, while stretching contributes negatively.

$L = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 - \frac{1}{2}k_1(q_1 - q_2)^2$.

The Euler-Lagrange equations use L to determine how the masses move.

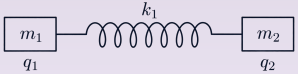
Hamiltonian mechanics

Describe the state using positions and momenta: $(q, p) \in T^*Q$.

$H = T + V$: it is the total kinetic and potential energy of the state.

$H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{1}{2}k_1(q_1 - q_2)^2$.

Hamilton's equations use H to determine how position and momenta evolve with time.



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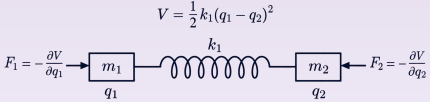
Instead of doing all that, we first encode the spring's position-force relation geometrically.

From potential to force



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Moving the masses changes this energy by dV_q, physically, the force covector is $-dV_q$

Thus the spring determines, at every position q, a particular covector dV_q.

Geometric structure

Collecting all position-covector pairs:

$$L_V = \{(q, dV_q) : q \in Q\} = \text{graph}(dV) \subseteq T^*Q.$$

This is a submanifold of the spring's phase space T^*Q.

It is **Lagrangian** because $\omega_{\text{can}}|_{L_V} = 0$ and $\dim L_V = \frac{1}{2}\dim T^*Q$.

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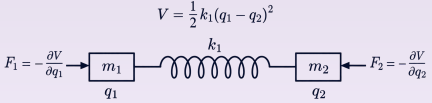

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We now know the data behind one spring. What happens when we join springs?

The BWY approach

A system is an **augmented span** $(X, F_X) \leftarrow (S, F_S) \rightarrow (Y, F_Y)$.

The span describes how the system connects; the augmentation carries its potential V or Hamiltonian H.

Systems are composed by identifying shared parts using a pullback.

Why is this complicated?

Lagrangian and Hamiltonian mechanics preserve different geometric structure, so BWY need different categories for them.

Neither category has the ordinary pullbacks needed to compose spans.

They therefore introduce generalized F-pullbacks and span-tightness.

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Given fixed component systems, we want to vary their interaction without redefining the systems themselves.

Systems have interfaces

Think of a system S as something carrying an interface I through which it may interact.
Two systems can first be placed side by side:

$$S_1 \otimes S_2.$$

At this stage, we have specified the components, but not how they are connected.

Interactions act on systems

An interaction $\mu : I \rightsquigarrow J$ specifies how an interface is reorganised.
It acts on the system:

$$S \triangleleft \mu.$$

So for the two springs, different choices μ_A, μ_B give

$$(S_1 \otimes S_2) \triangleleft \mu_A, \quad (S_1 \otimes S_2) \triangleleft \mu_B.$$

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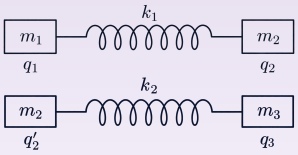
The systems



The two springs are unchanged.
Putting them side by side does not mean that $q_2 = q_2'$.

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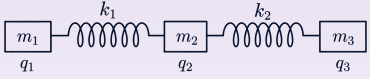

The choice that $q_2 = q_2'$ is separate data.

It acts on the two systems to produce the connected spring.

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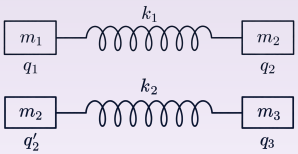

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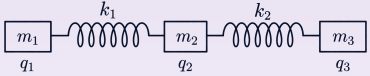

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V. Our construction: first, we regain the power of pullbacks!

We keep the geometry of classical mechanics, but move to a setting where pullbacks always exist.

Presymplectic geometry

A **presymplectic space** is a smooth space X equipped with a closed 2-form ω_X.
A map $f : (X, \omega_X) \to (Y, \omega_Y)$ is **presymplectic** when $f^*\omega_Y = \omega_X$.
So the map preserves the 2-form geometry.

Why a smooth topos?

Pullbacks of manifolds need not be manifolds.
A smooth topos $\mathcal{H}$ enlarges the smooth world so that pullbacks always exist.
We work in $\mathcal{C} = \mathcal{H}/\Omega^2_{\mathrm{cl}}$.
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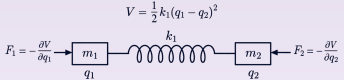

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Thus the spring determines $dV : Q \to T^*Q,\ q \mapsto (q, dV_q)$.

Why this slice helps

In $\mathcal{C} = \mathcal{H}/\Omega^2_{\mathrm{cl}}$, we have $(Q, 0)$ and $(T^*Q, \omega_{\mathrm{can}})$.
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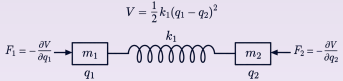

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Spans and pullbacks

Let $\mathcal{C}$ be a category with pullbacks.
A span

$$X \leftarrow M \rightarrow Y$$

composes with

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by pulling back over the shared interface Y:

$$X \leftarrow M \times_Y N \rightarrow Z.$$

Thus spans in $\mathcal{C}$ form the loose arrows of the double category $\mathrm{Span}(\mathcal{C})$.

Our presymplectic case

For

$$\mathcal{C} = \mathcal{H}/\Omega^2_{\mathrm{cl}},$$

all pullbacks exist since $\mathcal{C}$ is a topos.
Parallel composition is

$$(X, \omega_X) \otimes (Y, \omega_Y) = (X \times Y,\ \pi_X^* \omega_X + \pi_Y^* \omega_Y),$$

with unit $(1, 0)$.
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Libkind–Myers, Explication 6.4

Point $\mathrm{Span}(\mathcal{C})$ at $(1, 0)$.
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A **system** on I is a span

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What does the interaction do?

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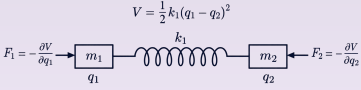
A single spring



The configuration space is $Q = \mathbb{R}^2$ and

$$V_1(q_1, q_2) = \frac{1}{2}k_1(q_1 - q_2)^2.$$

Take the phase-space interface (T^*Q, ω_{12}), where $\omega_{12} = dq_1 \wedge dp_1 + dq_2 \wedge dp_2$.
Define

$$x = dV_1 : Q \to T^*Q,$$

so $dV_1 = k_1(q_1 - q_2)(dq_1 - dq_2)$.
Thus x assigns to each configuration the covector describing the change of potential there, while its negative is the spring force.

The corresponding system

By Libkind-Myers Explication 6.4, we need a span in $\mathcal{C}$:

$$(1, 0) \xleftarrow{!} (Q, 0) \xrightarrow{x} (T^*Q, \omega_{12}).$$

The left leg is the unique map to $(1, 0)$, and $!^*0 = 0$.
For the right leg,

$$\begin{aligned} x^*\omega_{12} &= dq_1 \wedge k_1(dq_1 - dq_2) \\ &\quad + dq_2 \wedge [-k_1(dq_1 - dq_2)] \\ &= 0. \end{aligned}$$

Hence both legs are maps in $\mathcal{C}$, so the span is a system.

The left leg is trivial, but requiring it to be presymplectic forces the configuration-space form to be $0 : !^*0 = 0$.

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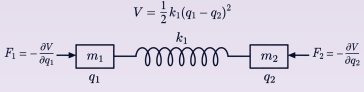

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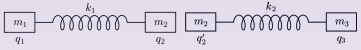
VI. Examples: placing springs side by side

Two independent springs



Let

$$V_1 = \frac{1}{2}k_1(q_1 - q_2)^2, \quad V_2 = \frac{1}{2}k_2(q_2' - q_3)^2.$$

The corresponding systems are

$$s_{12} : (1,0) \leftarrow (\mathbb{R}^2, 0) \xrightarrow{dV_1} (T^*\mathbb{R}^2, \omega_{12}),$$

$$s_{23} : (1,0) \leftarrow (\mathbb{R}^2, 0) \xrightarrow{dV_2} (T^*\mathbb{R}^2, \omega_{23}).$$

Tensor product places them in parallel, without identifying any coordinates.

The corresponding system

Their configuration spaces combine to $\mathbb{R}^4$, with coordinates (q_1, q_2, q_2', q_3).
Set

$$W = V_1(q_1, q_2) + V_2(q_2', q_3).$$

Then

$$s_{12} \otimes s_{23} = \left[(1,0) \leftarrow (\mathbb{R}^4, 0) \xrightarrow{dW} (T^*\mathbb{R}^4, \omega_4)\right].$$

Under $T^*(\mathbb{R}^2 \times \mathbb{R}^2) \cong T^*\mathbb{R}^2 \times T^*\mathbb{R}^2$,

$$dW = (dV_1, dV_2).$$

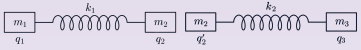
Since each dV_i preserves the chosen 2-forms,

$$(dW)^*\omega_4 = (dV_1)^*\omega_{12} + (dV_2)^*\omega_{23} = 0.$$

Hence this is again a system. Importantly, q_2 and q_2' remain independent.

VI. Examples: placing springs side by side

Two independent springs



Let

$$V_1 = \frac{1}{2}k_1(q_1 - q_2)^2, \quad V_2 = \frac{1}{2}k_2(q_2' - q_3)^2.$$

The corresponding systems are

$$s_{12} : (1,0) \leftarrow (\mathbb{R}^2, 0) \xrightarrow{dV_1} (T^*\mathbb{R}^2, \omega_{12}),$$

$$s_{23} : (1,0) \leftarrow (\mathbb{R}^2, 0) \xrightarrow{dV_2} (T^*\mathbb{R}^2, \omega_{23}).$$

Tensor product places them in parallel, without identifying any coordinates.

The corresponding system

Their configuration spaces combine to $\mathbb{R}^4$, with coordinates (q_1, q_2, q_2', q_3).
Set

$$W = V_1(q_1, q_2) + V_2(q_2', q_3).$$

Then

$$s_{12} \otimes s_{23} = \left[(1,0) \leftarrow (\mathbb{R}^4, 0) \xrightarrow{dW} (T^*\mathbb{R}^4, \omega_4)\right].$$

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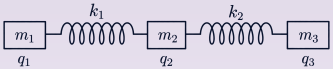
Hence this is again a system. Importantly, q_2 and q_2' remain independent.

VI. Examples: joining the middle masses

Joining the middle mass



Start with

$$(1,0) \leftarrow (\mathbb{R}^4, 0) \xrightarrow{dW} (T^*\mathbb{R}^4, \omega_4),$$

where q_2, q_2' are independent.
Define

$$\delta : \mathbb{R}^3 \to \mathbb{R}^4, \quad \delta(q_1, q_2, q_3) = (q_1, q_2, q_2, q_3).$$

This imposes $q_2 = q_2'$.
It gives the interaction

$$\mu_\delta : (T^*\mathbb{R}^4, \omega_4) \leftarrow (C_\delta, \omega_{C_\delta}) \to (T^*\mathbb{R}^3, \omega_3),$$

where $C_\delta = \delta^* T^*\mathbb{R}^4$.

The corresponding system

A point of C_δ is (q, α), with $\alpha \in T^*_{\delta(q)}\mathbb{R}^4$.
The left leg sends $(q, \alpha) \mapsto \alpha$; the right leg sends $(q, \alpha) \mapsto \delta_q^* \alpha$. So forces are pulled back along the constraint.
Acting by pullback gives

$$(s_{12} \otimes s_{23}) \triangleleft \mu_\delta \cong \left[(1,0) \leftarrow (\mathbb{R}^3, 0) \xrightarrow{d(W \circ \delta)} (T^*\mathbb{R}^3, \omega_3) \right].$$

Indeed, the apex is $\mathbb{R}^3$, and

$$\delta^*(dW) = d(W \circ \delta).$$

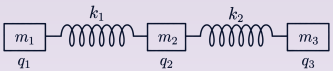
Finally,

$$W \circ \delta = \frac{1}{2} k_1 (q_1 - q_2)^2 + \frac{1}{2} k_2 (q_2 - q_3)^2.$$

Thus the interaction gives the force relation for two springs whose middle masses move together.

VI. Examples: joining the middle masses

Joining the middle mass



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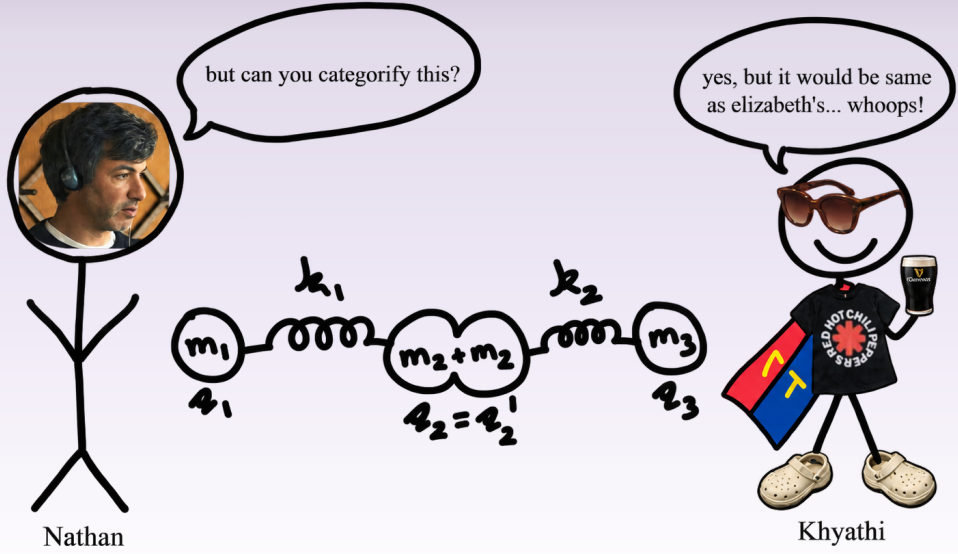
VII. Motivating future work



But why is that the case?

- Our systems are defined using dV, which captures the position-force relation determined by the potential, but contains no information about mass.
- Consequently, our construction cannot distinguish a single middle mass m_2 from two rigidly joined masses of total mass $2m_2$, both yield the same dV.
- To capture this distinction, we need to incorporate kinetic energy through Hamiltonians $H = T + V$, and extend our theory of interactions to account for the resulting dynamics.

VIII. Acknowledgements

This work was carried out during my Summer Research Associateship at the Topos Institute, under the mentorship of Brendan Fong and Kevin Carlson. I am grateful to David Jaz Myers for motivating the problem, and to both him and Jason Brown for patiently answering qustions. I also thank everyone at the Topos Institute for their warmth and kindness throughout the summer. Special thanks to Henry and Piash for letting me borrow their iPad and laptop respectively :)

IX. Conclusion

What we built

- A spring gives a system

$$(1,0) \leftarrow (Q,0) \xrightarrow{dV} (T^*Q, \omega).$$

- Tensor places systems side by side.
- Interactions are spans between interfaces.
- Acting by an interaction means pulling back.

So the components stay fixed while the assembly changes.

What comes next

- The present construction records geometry.
- It handles constraints such as

$$q_2 = q_2'.$$

- But it does not yet record inertia.
- Hamiltonian flavouring should add the missing mass-sensitive dynamics.

X. References

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Thank You!